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ANALYSIS OF THE RESULTS OF SIMULTANEOUS MEASUREMENTS OF ELECTRON CONCENTRATION IN THE IONOSPHERE WITH THE HELP OF IONOSPHERIC STATIONS AND ROCKETS

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K. I. Gringauz
G. L. Gdalevich

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The integral formula lying at the basis of all methods for the obtention of electron concentration in height from high-frequency characteristics is as follows:

$$H_{д} = \int_0^H \frac{dh}{n}, \qquad (1)$$

where $H_{Д}$ is the "acting" reflection height, H is the true height of reflection, n is the index of refraction for radiowaves with the frequency ω.

There are two groups of methods for finding $n_e(h)$, which is the altitude distribution of electron concentration n_e by high-frequency characteristics: those called "comparison methods" or "matching", and methods based upon the solution of an integral equation.

The utilization of the comparison methods implies the assignment of s series of laws $n_e(h)$ expressed analytically. At substitution of each of such $n_e(h)$ distribution in (1), the equation is reduced to taking the integral. The experimental hf-characteristic is compared with the family of high-frequency characteristics, from which that closest to the experimental is selected. The law of $n_e(h)$ distribution utilized in the computation of this characteristic is closest to the experimental and is taken as the true law.

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* Analiz rezul'tatov odnovremennykh izmereniy elektronnoy kontsentratsii v ionosfere s pomoshch'yu ionosfernykh stantsiy i raket.

The second group of methods is linked with the "precise" solution of the integral equation (1). At the same time the solution will be single-valued only in the case when the $n_e(h)$ distribution will be a monotonic function. In substance the monotony of $n_e(h)$ is as much necessary also for the possibility of utilzing comparison methods (for solution's single-valuation).

Since prior to the beginning of rocket investigations of the ionosphere, the opinion on its laminar structure prevailed (i. e. on the essentially nonmonotonic character of the altitude distribution of $n_e(h)$), it appears that for the same reason the methods consisting in the solution of the integral equation did not receive much application in spite of the fact that they were known as far back as 1937 [1]. The utilization of these methods was also made practically difficult because of the absence of electronic computers at that time for computations that are decidedly cumbersome.

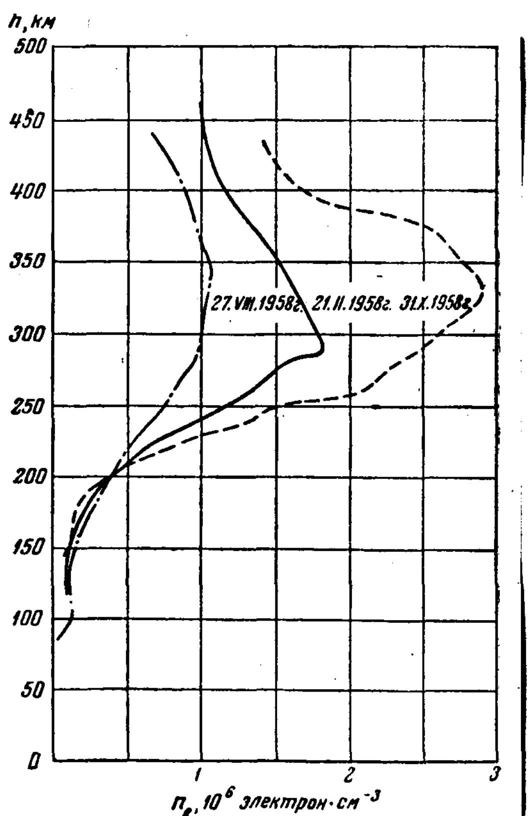
When the rocket investigations of the ionosphere have shown that the departures of the function $n_e(h)$ from monotony are in reality insignificant, the interest as regards these methods was aroused, and they became the object of broader applications [2 — 5].

We must however note, that thus far only one work is known to us [2], where a comparison is made of the results of $n_e(h)$ determination by the method of integral equation using the experimental high-frequency characteristic, with the distribution of $n_e(h)$ to the 200 km altitude, simultaneously obtained with the help of rockets.

Meanwhile it is obvious that for the final appraisal of the applicability and precision of $n_e(h)$ determination by hf characteristics, such comparisons are necessary, and they must be conducted more than once to the desirable height of the prinicpal ionization maximum in the ionosphere. Attempte are made in the present paper to partly fill that gap.

1. COMPUTATION OF HIGH-FREQUENCY CHARACTERISTICS ON THE BASIS OF n_e (h) DISTRIBUTIONS OBTAINED WITH THE AID OF ROCKETS.

Fig. 1 presents the results of n_e (h) measurements by means of rockets, conducted in the morning and in daytime [6]. As may be seen from Fig. 1, the portions of the curves n_e (h) situated below the prinicpal ionization maximum show only insignificant departures from monotony (in the form of small maxima). We may however show, that for such altitude distributions of n_e (h), breaks in height reaching about 100 km must be observed at hf characteristics obtained by ionospheric stations. (The existence of such breaks contributed in its time to shaping up the concept about the "laminar" or "stratified" ionosphere). In order to simplify the computation, let us show this for the case when the dependence of the index of refraction n on the magnetic field and on frequency of electron collisions, is not taken into account, i. e., we shall take the expression for n in the form:



electron cm^{-3}

Fig. 1

$$n = \sqrt{1 - \frac{4\pi e^2 n_e(h)}{m\omega^2}}. \quad (2)$$

Here e, m respectively are the charge and the mass of the electron, is the radiowave frequency, and n_e (h) is the electron concentration at the height h.

Breaking up the curve of electron concentration distribution obtained with the help of rocket measurements in a series of portions on each of which $n_e(h)$ may be considered a linear function h, and substituting n in (1) as a function h in the form (2), we may take the integral (1) and easily obtain at the given frequency for the "acting" thickness of the i-th portion :

$$H_{\text{д}\,i} = \frac{2H_i}{x_i - x_{i-1}} \left(\sqrt{1 - x_{i-1}} - \sqrt{1 - x_i}\right), \qquad (3)$$

where H_i is the true height of the i-th portion, $x_i = \frac{4\pi e^2}{m\omega^2} n_i(h)$, $x_{i-1} = \frac{4\pi e^2}{m\omega^2} n_{i-1}$, n_{i-n}, n_i respectively are the concentration at the beginning and at the end of the portion.

The computation of the "acting" height for the frequency is made in the following manner. The curve of distribution of electron concentration is broken into a series of portions to the critical concentration n_k, at which reflection of radiowaves of frequency takes place. The "acting" thickness is determined over each portion by the expression

$$H_{\text{д}} = h_0 + \sum_i \frac{2H_i}{x_i - x_{i-1}} \left(\sqrt{1 - x_{i-i}} - \sqrt{1 - x_i}\right), \qquad (4)$$

where h_o is the height of the lower boundary of the ionosphere.

In the case when electron concentration is constant at the i-th portion, we obtaine directly from (1)

$$H_{\text{д}\,i} = \frac{H_i}{\sqrt{1 - \frac{4\pi e^2}{m\omega^2} n_e}}. \qquad (5)$$

Such was the way of computing the values of the "active" heights for a series of frequencies. The coincidence of these values allow to construct a hf characteristic for each of the $n_e(h)$ distributions brought out in Fig. 1.

In order to compare the thus computed hf characteristics with the real characteristics obtained at the ionospheric station situated in the region of rocket launching at time of latters' flight, a series of points of computed hf characteristics were plotted on the photographs of experimental hf characteristics (see Fig. 2, 3, 4). It may be clearly seen on the photographs that the computed characteristics are close to the real, and in particular, that they too have breaks by altitude.

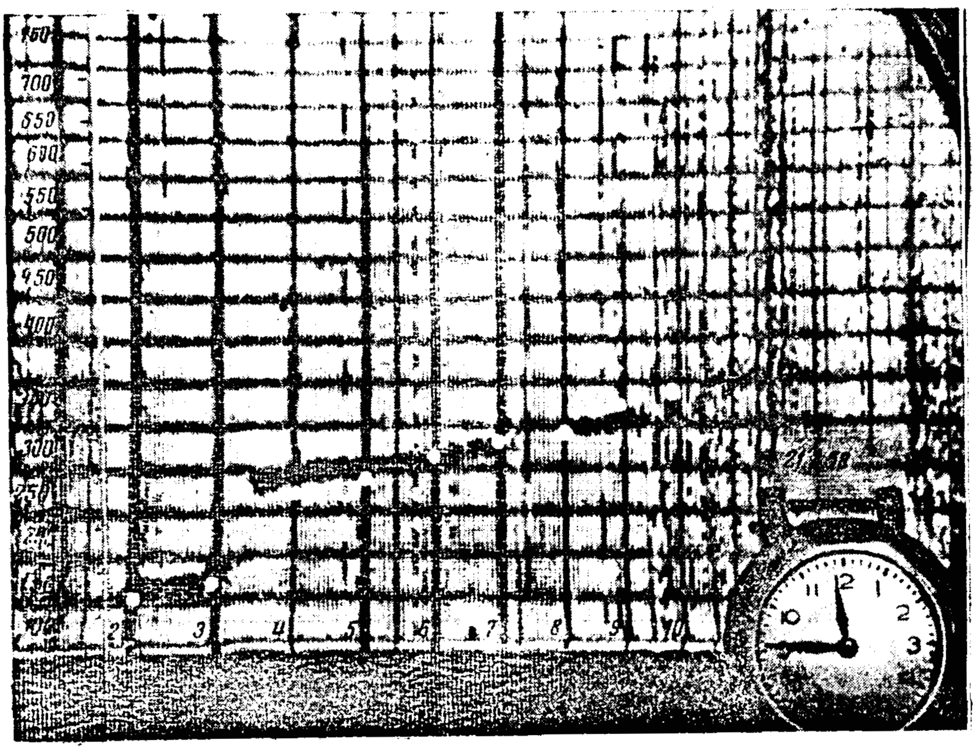


Fig. 2

It must be noted that a break in the region $f = 3$ mc/s corresponding to the so-called E_2 -layer exists on the experimental characteristic obtained on 27 July 1958. This break is absent on the computed hf characteristic, which apparently may be explained by the fact, that the curves n_e (h) utilized at computations, are smoothened within the 5% limits [6].

The closeness of the hf characteristics obtained by rocket measurements of the distribution of electron concentration to those obtained by sounding methods from the ground, corroborates the realibiliy of rocket measurements, and the coreectness of conc,usions that the ionosphere region in the 100 — 300 km altitude range constsitutes a medium with a near monotonic rise of electron concentration, without dividing into separate layers.

This justifies the assumption about the presence of a single (principal) maximum of n_e, in the making at computations of the distribution of electron concentration on the basis of high-frequency characteristics.

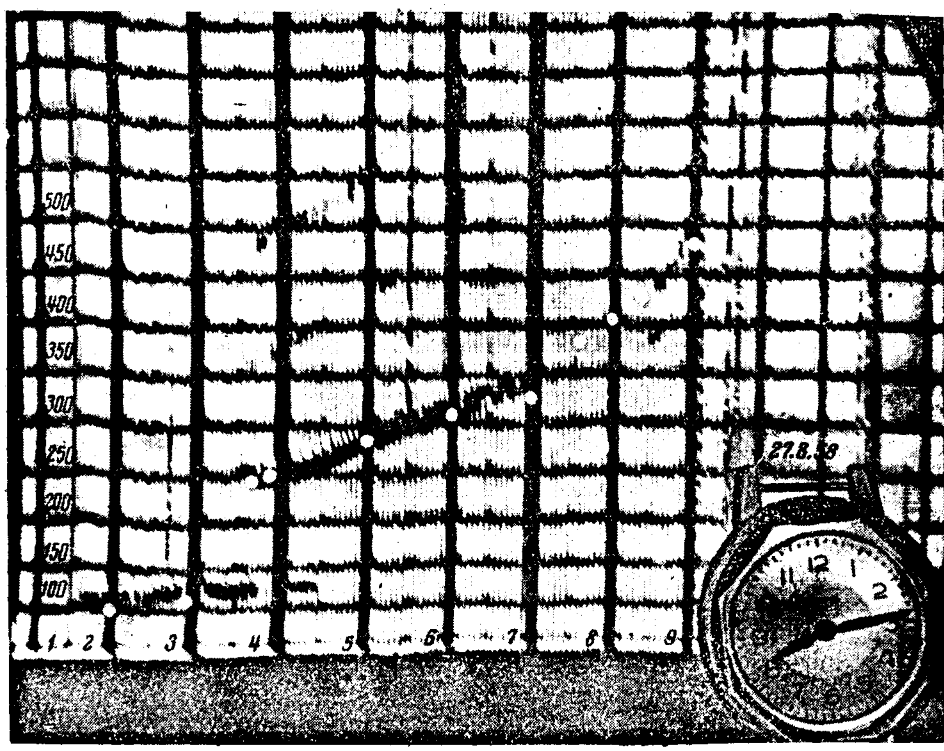
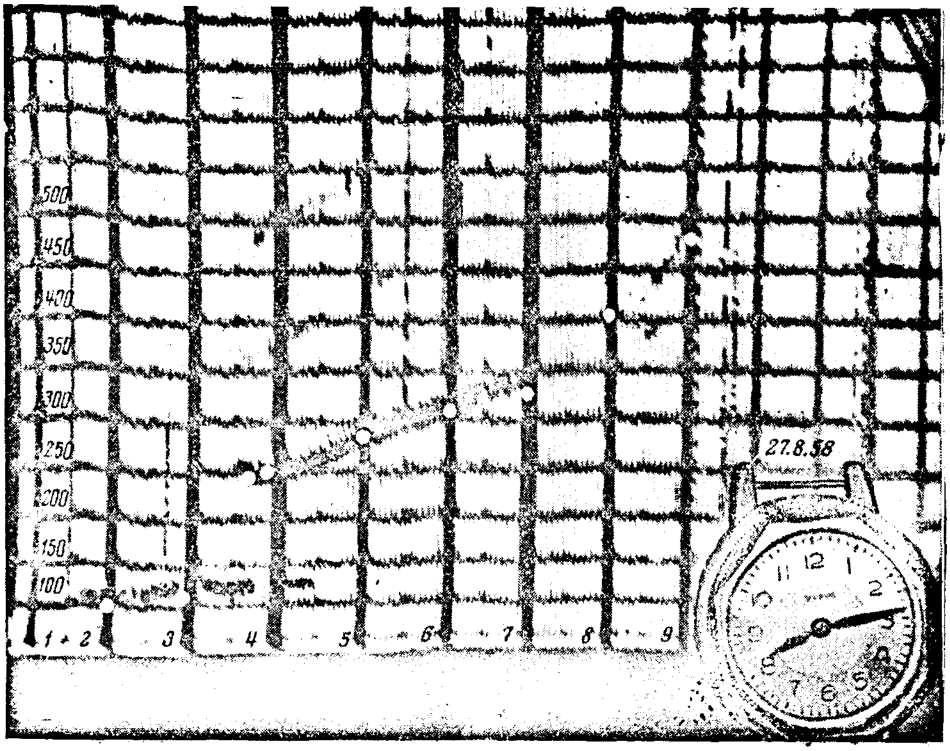


Fig. 3

2. COMPARISON OF n_e (h) DISTRIBUTIONS COMPUTED ON THE BASIS OF HIGH FREQUENCY CHARACTERISTICS, WITH THOSE OBTAINED WITH THE AID OF A DISPERSION INTERFEROMETER.

The computation of the altitude distribution of electrons by hf characteristics was conducted by the method of integral equation solution with the help of Shinn-Kelso coefficients, taking into account the effect of the Earth's magnetic field.


500
450
400
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3
4
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8
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27.8.58

Fig. 3.

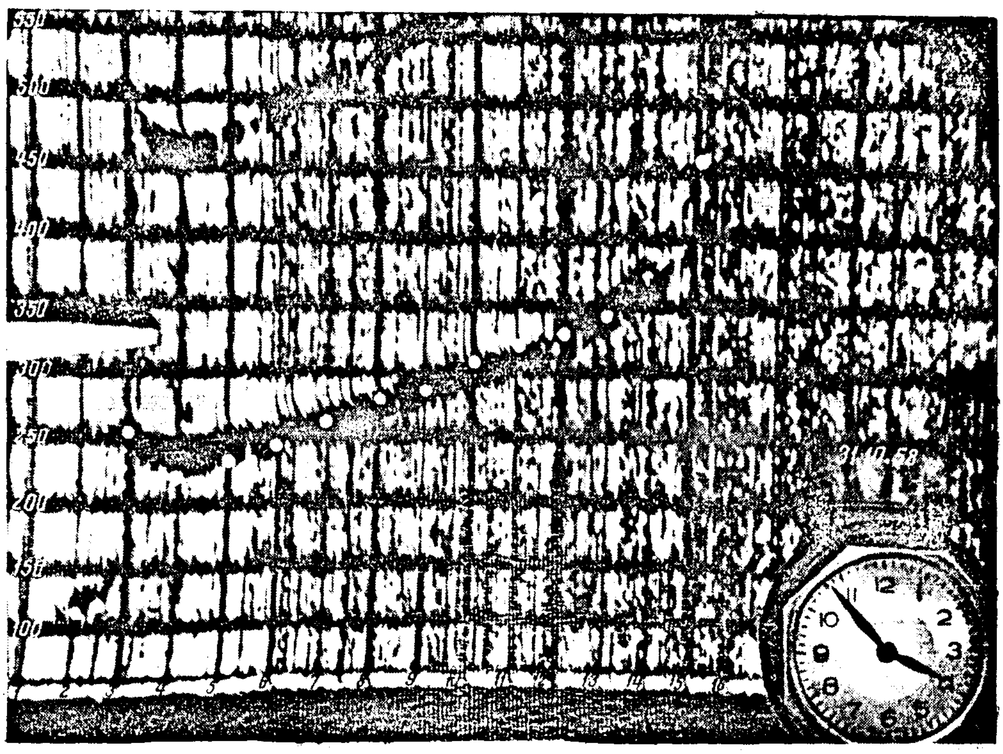
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Fig. 4.

Utilizing that method for the solution of (1), $H_Д$ is considered to be the assigned function of frequency $H_Д = F(f)$, determined by the ionospheric high-frequency characteristic.

If we neglect th e effects of the Earth's magnetic field and of electron collisions with neutral particles, then (1) is reduced to the Abel integral equation heving the solution (see [7])

$$H_{f_0} = \frac{2}{\pi}\int_0^{f_0} \frac{H_Д(f)\,df}{\sqrt{f_0^2 - f^2}}, \qquad (6)$$

where

$$f_0^2 = \frac{e^2 n_e}{\pi m}, \quad f^2 = \frac{\omega^2}{4\pi^2}.$$

It is difficult to make use of formula (6) in practice, for it is made difficult by the fact that the integral entering in in it is improper. In order to make possible the computation of the integral, a bew variable $\theta = \arcsin \frac{f}{f_0}$. Then (6) takes the form

$$H(f_0) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} H_Д(f_0 \sin\theta)\,d\theta. \qquad (7)$$

The integration of (7) is carried out numerically, by subdiving the curve $H_Д(f_0 \sin\theta)$ into a series of steps of equal windth by θ [8]. At the same time the quantities $\frac{f}{f_0}$, determining the selected steps, are called the Kelso coefficients.

In order to take into account the effect of the magnetic field, Shinn introduced modified coefficients [9], called Shinn — Kelso coefficients. When utilizing these coefficients, the true altitude, corresponding to f_0, is given by the correlation

$$H(f_0) = \frac{1}{N}\sum_{i=1}^{N} H_Д(f_i), \qquad (8)$$

where N is the number of steps, and f'_i are determined by the Shinn-Kelso coefficients.

Taking into account that angle of inclination and the magnitude of magnetic field strength in the region where the rockets of the USSR Academy of Sciences were launched, are close to the corresponding quantities at the location of the English station Slow we utilized the Shinn-Kelso coefficients during the computation for five points, brought out in [4] * The computation was conducted with an interval of 0.2 mc/s according to formula

$$h(f_k) = \frac{1}{5}\sum_{1}^{5} h'(f_i),$$

where $h(f_k)$ is the true reflection height for the frequency f_k at which the electron concentration (n_e electron cm^{-3}) $= 1.24 \cdot 10^{-4}$ f_k mc/s, $h'(f_i)$ is the "acting" height for the frequency f_i, which is found from f_k with the help of Shinn-Kelso coefficients.

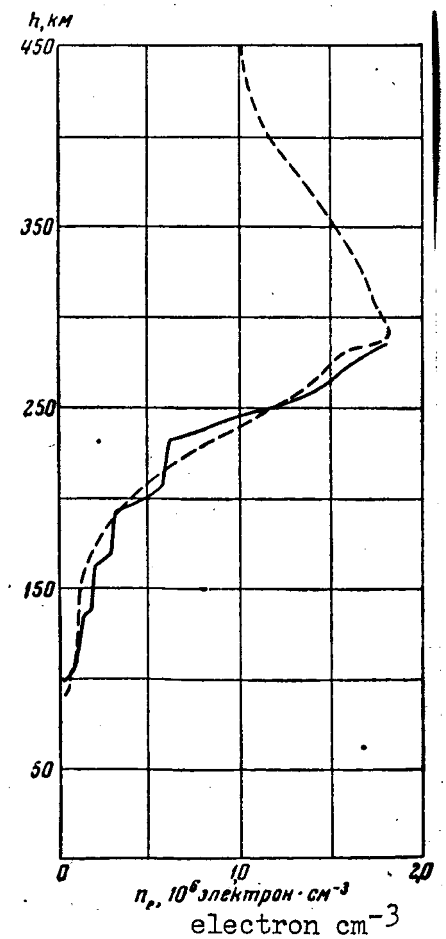

Fig. 5.

The results of computations are presented in Figures 5, 6 and 7, in which dashed lines represent the distributions of electron concentration obtained with the aid of rocket measurements [6]. It may be seen from Fig. 5, 6 and 7 that the correspondence is fairly good.

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* After completion of computations, work in reference [10] was published. In it are presented coefficients for finding the distribution of concentrations by hf characteristics (accounting the magnetic field. The coefficients are given for all stations with magnetic inclination $\leqslant 80^\circ$.

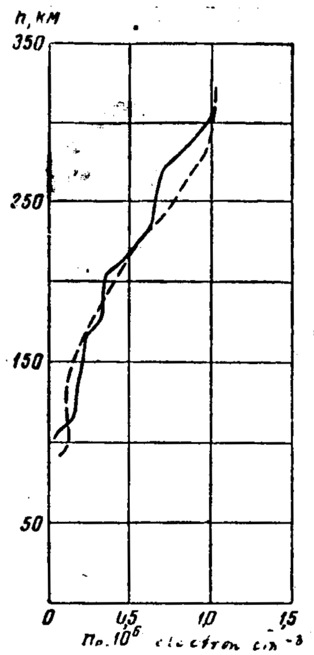

Fig. 6

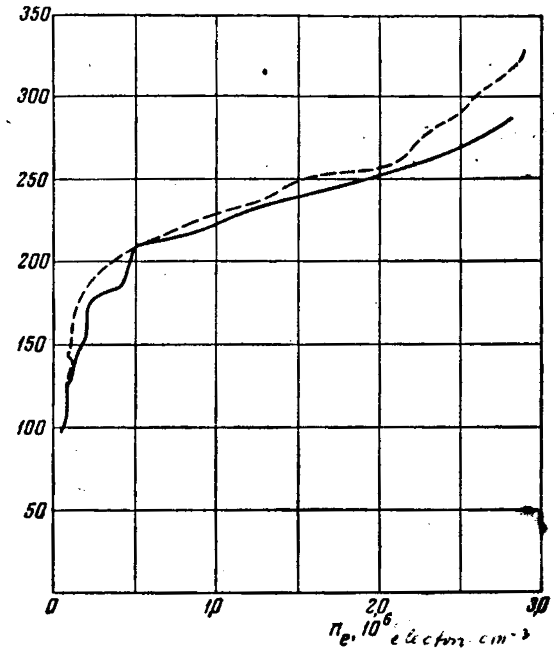

Fig. 7

The divergences between the computed and the experimental curves of distribution of electron concentration may be explained by the following causes :

1. Utilization of a small number of coefficients (five).

2. Inaccuracy in the computation of "acting" heights and frequencies by hf characteristics, on account of errors originating in the instrumentation, and because of signal widening at their reflection.

3. Absence of frequencies below 1.5 mc/s in the band of the utilized ionospheric station (as a result of which the retardation in the very lowest layers is not taken into account).

4. The presence of some minor departures from the monotonic course of the concentration between the E- and F-layers.

5. Averaging (within the 5% limits) obtained with the aid of the dispersion interferometer, of the distribution curve.

Disregarding and not analyzing the errors occurring as a consequence of each of these causes, we may estimate the magnitude of the error on the basis of comparison of the results. The maximum divergence when utilizing the 5 coefficients does not exceed 15%.

It must be noted, that in all cases in which comparison was made, the sporadic E_s- layer was absent. If such layer is present, the errors in the computation of n_e (h) distribution by hf characteristics may be great.

The above considerations show that the obtention of electron distributions in height to the altitude of the F_2- layer maximum from hf characteristics is possible with relatively small errors by using the Shinn-Kelso coefficients.

It is thus desirable that at least a small part of the results of observations by the ionospheric stations network be presented in the form of n_e (h) distribution. This would permit the accumulation of data on n_e (h) variations with the time of the day and year, and would contribute to the clarification of a series of questions concerning ionosphere physics.

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***** E N D *****

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